

# A Model for Vehicle-to-Infrastructure Communications in Urban Environments

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**Abstract**—Dependable vehicle-to-infrastructure (V2I) communication links for intelligent transportation systems are aimed at improving traffic safety and efficiency. The future integration of such cooperative systems, in particular in urban environments with challenging propagation conditions, requires adequate deployment of roadside units (RSU). To analyze the joint impact of typical urban characteristics and RSU deployment conditions on the quality of IEEE 802.11p V2I communications, an extensive measurement campaign was performed in Bologna as part of the European FP7 project iTETRIS. Based on the measured data, we identify specific street layouts and propagation impairments, and we estimate the parameters of a previously introduced range-dependent modified Gilbert model. The resulting set of model parameters enables the generation of realistic packet error patterns for any comparable urban environment.

## I. INTRODUCTION

Cooperative vehicular networks enable a wide variety of services which improve traffic safety and efficiency. The performance of such networks depends on multiple factors, including the propagation environment, the transmission parameters, the relative position and velocity of communication nodes, and the traffic density. Therefore, to carry out efficient and reliable system design, the performance impact of these factors should be taken into account.

Communication performance can be analyzed by means of field testing campaigns. In [1], the performance of the IEEE 802.11p standard was evaluated in highway scenarios. It was shown that vehicle-to-infrastructure (V2I) communication performance is strongly influenced by the presence of large objects blocking the line of sight (LOS). In [2], it was shown that longer packets are more suitable than higher data rates for increasing the throughput of IEEE 802.11p-based V2I communications. The influence of the roadside unit (RSU) antenna type on the system performance was analyzed in [3]. It was shown that directional antennas are more suitable for IEEE 802.11p-based V2I communications than omnidirectional antennas. However, most studies in this context focus on highway environments and thus the analysis of urban environments deserves particular attention. Unfortunately, the costs of a measurement campaign can be prohibitive and therefore field tests are often avoided.

Consequently, the design and optimization of vehicular networks is to a large extent based on simulations, which require

realistic models for the radio links. There exist different ways of modeling the propagation channel ranging from ray-tracing and replay models to stochastic channel models. Ray-tracing models yield an excellent approximation of the real-world propagation environment at the cost of high computational complexity, see, e.g., [4]. For replay models (e.g., [5]), the vehicular transmission is measured in a realistic environment and the result is directly used as an input for a simulator. Stochastic models, which describe the wireless channel from a macroscopic point of view, provide a balance between accurate modeling of the radio link and computational complexity. In this context, [6] suggests a random birth-death process was proposed to model the individual multipath components of time-variant radio channels. The study in [7] proposed a context-based performance model for V2I networks which takes propagation conditions, traffic density, and RSU locations into account. Following the idea of parsimonious packet-level performance modeling, a computationally inexpensive approach to model packet errors for V2I communications was introduced in [8].

In this contribution, we propose a modification of the range-dependent modified Gilbert model of [8]. The proposed modification results in a simple model that allows us to account for realistic urban street layouts and propagation impairments. The data we use has been obtained through real-world measurements performed at 5.9 GHz in an urban environment. We next give a brief overview of this measurement campaign (Sec. II, cf. [9] for details) and our modeling approach (Sec. III).

## II. MEASUREMENT CAMPAIGN SETUP

The parametrization of the extended range-dependent modified Gilbert model presented in this paper is based on real-world packet error traces. The measurement data was obtained through an extensive field testing campaign that has been conducted as part of the European FP7 project iTETRIS [10]. Measurements were performed on 20 km of urban road network in Bologna, Italy. The campaign included 22 carefully selected RSU locations (see [9] for details).

Throughout the field testing campaign, a vehicle with an on-board unit (OBU) was used as a receiver and two portable RSUs placed next to the road were used as transmitters.

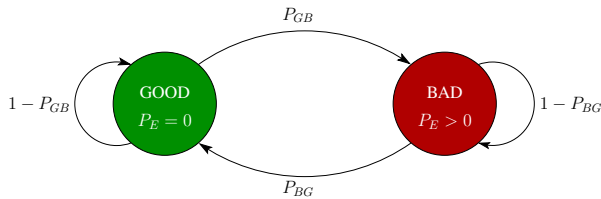


Fig. 1: Schematic illustration of the Gilbert model.

All communication units were equipped with IEEE 802.11p prototypes controlled by a standard laptop.

As OBU antenna, a Nippon omni-directional antenna with 0 dBi gain was used. The antenna was placed on the rooftop of the vehicle and was connected to the receiver using 3 m LMR240 cable ( $\sim 3$  dB cable loss). As RSU antenna, an ECO12-5800 omni-directional antenna with 12 dBi gain was used. The antenna was placed on top of a portable pneumatic telescopic mast and was connected to the transmitter using 14 m LMR400 cable ( $\sim 5$  dB cable loss). For the measurements presented in this contribution, the transmit power of the IEEE 802.11p prototype was set to 20 dBm. Thus, the equivalent isotropically radiated power of the transmitter was 27 dBm. The RSU antenna height was 6.5 m, which is about the same as the height of a traffic light.

The measurements were performed on the IEEE 802.11p control channel in the frequency range of 5.895 – 5.905 GHz. The RSUs were configured to broadcast 10 packets per second with data rate of 6 Mbit/s, corresponding to QPSK modulation with rate-1/2 channel coding. The length of the transmitted packets was 126 Bytes. The payload of each packet included a timestamp, a unique packet ID, as well as the position and ID of the transmitter. All correctly received packets were recorded by the receiver together with the information about the vehicle’s speed, heading, and position. Additionally, the transmitter stored all transmitted packets.

### III. PRELIMINARIES OF MODELING APPROACH

In our previous work [8], we proposed a range-dependent modified Gilbert model as a computationally efficient method for generating realistic V2I packet error patterns. This model is basically an extension of a simple two-state hidden Markov model introduced by Gilbert [11]. As shown in Fig. 1, Gilbert’s model is fully described by only three parameters: the transition probability from the bad state to the good state,  $P_{BG}$ , the transition probability from the good state to the bad state,  $P_{GB}$ , and the error probability in the bad state,  $P_E$ .

When modeling real-world V2I channels, Gilbert’s two-state model cannot reproduce the distance-dependent link quality with sufficiently high accuracy. In [8], we thus proposed to divide the measured error patterns into  $N$  parts, corresponding to  $N$  disjoint distance intervals of the same length (henceforth called *granularity*). The model parameters are then estimated for each interval using the Baum-Welch algorithm [12]. Once the model parameters for all  $N$  intervals are estimated, they are combined to form a range-dependent modified Gilbert model [8]. This model retains all properties of the original Gilbert

model, except for the fact that the model parameters change depending on the transmitter-receiver distance.

The granularity of the range-dependent modified Gilbert model constitutes a trade-off between accuracy and complexity (note that the Gilbert model corresponds to the special case  $N = 1$ ). In [8], we showed that an acceptable level of accuracy can only be achieved by estimating the model parameters with granularities  $\leq 10$  m. However, small granularities lead to a considerable increase of the number of intervals, thereby increasing the computational cost of the model. In order to ensure high accuracy while keeping the complexity low, a vector quantization of the parameters was proposed in [13].

The joint quantization of the parameters does not only reduce the model dimension, but it also allows us to associate each quantization level to a certain communication quality. To model the packet error performance, we require just a single set of parameters for each quantization level. A reasonable choice for the number of quantization levels was found to be  $K = 3$  (cf. [13]). Hence, we suggest to divide the coverage range into three regions according to the performance: high quality communication ( $Q_1$ ), intermediate ( $Q_2$ ), and unreliable ( $Q_3$ ) communication. Moreover, the quantized model parameters capture the influence of realistic impairment factors that affect the propagation conditions. For highway environments the most common impairments were found to be highway overhead signs and overpasses (cf. [13, Fig. 3]).

In order to create a realistic communication model, typical urban street layouts and propagation impairments need to be identified and classified. To this end, we thoroughly analyzed the urban environment in the city center of Bologna. We next present the considered street layouts (Sec. IV-A) and propagation impairments (Sec. IV-B).

## IV. STREET LAYOUTS AND IMPAIRMENTS FOR URBAN ENVIRONMENTS

### A. Street Layouts

Based on our observations, the city center of Bologna is sufficiently well represented by the following four types of street layouts:

- Layout 1: street surrounded by the buildings from both sides.
- Layout 2: street surrounded by the buildings from one side and by vegetation from the other side.
- Layout 3: street surrounded by vegetation from both sides.
- Layout 4: open area street.

Due to space constraints, we focus on the first two street layouts. An example of layout 1 is shown in Fig. 2(a). This layout is applicable to both one-way and two-way streets with two or three lanes in each direction. The type and the exterior material (brick, glass, metal, etc.) of the surrounding buildings on both sides of a street are not covered by our simple model. The only constraint is that the height of the buildings is  $\geq 6.5$  m, corresponding to the height of the transmit antenna in our measurements.

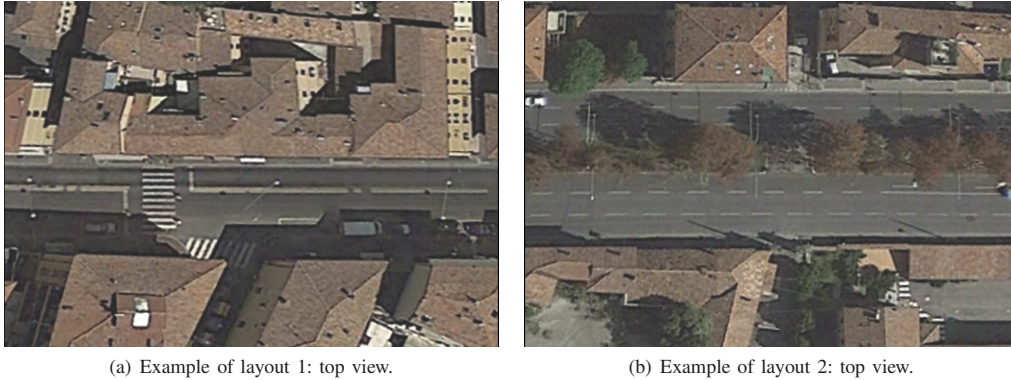


Fig. 2: Types of urban street layouts (source: Google earth).

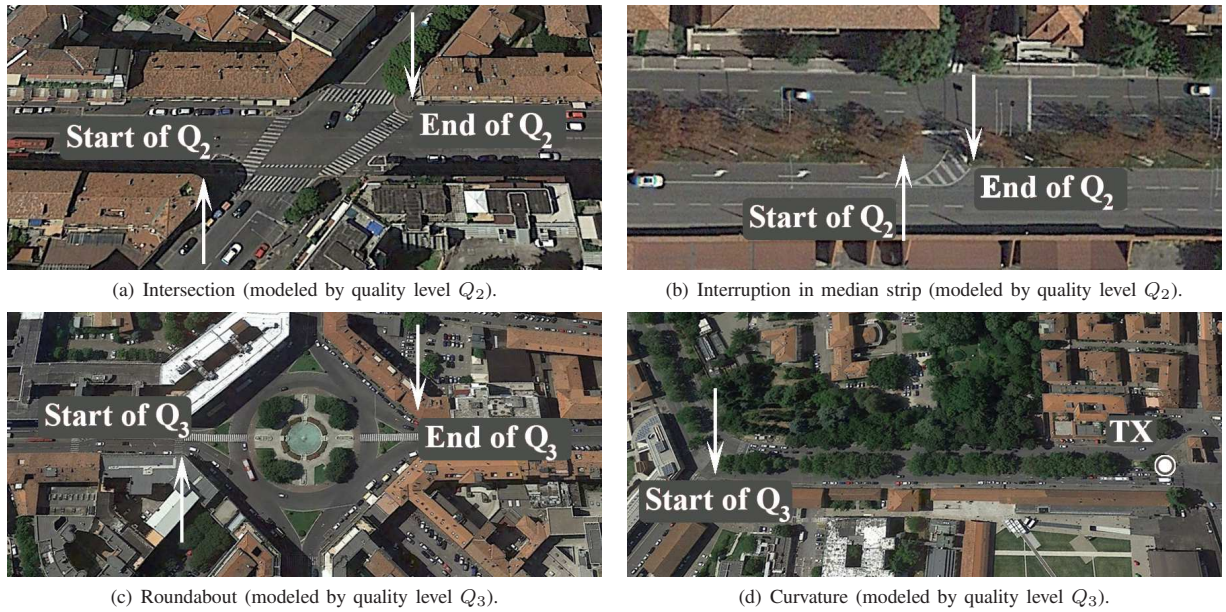


Fig. 3: Types of urban impairments (source: Google earth).

The second layout is applicable for both one-way and two-way streets. Since one-way streets with this layout are found less frequently, we focus on two-way streets. In Fig. 2(b), a two-way street is surrounded by the buildings from both sides and the driving directions are separated by a median strip covered with trees. In the case of a one-way street, this layout can be applied for streets surrounded by buildings from one side and by trees from the other side. For the sake of simplicity, the density and type of vegetation (trees, bushes, etc.) is not covered by our model.

### B. Impairments

The communication performance is clearly distance dependent with decreasing communication quality as the distance to the transmitter increases. Hence, it makes sense to assume that the quality levels appear in descending order. However, certain environmental factors may deteriorate the packet error performance, irrespective of the distance between transmitter

and receiver. In order to account for the performance loss due to environmental impairing factors, the model parameters need to be adjusted accordingly. We note that the range within which the model parameters are changed is determined by the largest dimension of the impairment in the driving direction plus 5 m on either side. In what follows, we introduce four types of impairments that were identified for the two urban street layouts defined in Sec. IV-A.

**Street intersection** (cf. Fig. 3(a)). This type of impairment occurs when two or more roads either meet or cross each other. A street intersection in close vicinity to the RSU reduces the communication performance. To model the influence of the street intersection, we propose to change the model parameters such that  $Q_1$  (high communication quality) is replaced by  $Q_2$  (intermediate communication quality) and  $Q_2$  is replaced by  $Q_3$  (unreliable communication). The quality level  $Q_3$  remains unchanged.

**Interruption in the median strip** (cf. Fig. 3(b)). This type of impairment is only applicable to the street layout 2 and occurs most frequently in connection with an intersection or a turnaround. Median strip interruptions in close vicinity to the RSU result in a slight performance degradation, while the performance impact at larger distances is negligible. Therefore, we model the influence of an interruption in the median strip by changing the model parameters from  $Q_1$  to  $Q_2$  within the high quality communication range. The model parameters remain unchanged in the other communication ranges.

**Roundabout** (cf. Fig. 3(c)). A roundabout is an intersection-like impairment which usually yields a loss of LOS connection to the transmitter. Therefore, the quality of communication is significantly degraded while the vehicle is in the roundabout. To account for this impairment, we propose to change the model parameters within the high and intermediate quality ranges to  $Q_3$ . We assume that after passing the roundabout, the vehicle continues driving in the same direction as before.

**Curvature** (cf. Fig. 3(d)). This impairment is used whenever the vehicle changes its heading by more than  $20^\circ$ . Measurement results show that if a curvature occurs, the communication quality is significantly deteriorated and eventually breaks down within 100m after the turn. To model this impairment, the model parameters are changed to  $Q_3$ . Furthermore, the coverage range ends 100m after the turn, irrespective of the distance between the transmitter and the the curvature.

## V. RESULTS

### A. Model Parameters

To obtain the model parameters we first divided the measured packet error traces into  $N$  intervals with a granularity of 5m. Next, the model parameters  $P_{GB,n}, P_{BG,n}, P_{E,n}$ ,  $n = 1, \dots, N$ , are estimated for each interval by applying the Baum-Welch algorithm to a sufficiently large number of measurement repetitions. Subsequently, the vectors  $\mathbf{p}_n = (P_{GB,n}, P_{BG,n}, P_{E,n})^T$ ,  $n = 1, \dots, N$ , are quantized with  $K$  quantization levels (see [13] for details). This vector quantization reduces the total number of model parameters from  $3N$  to  $3K$  and therefore entails a significant complexity reduction (note that  $N \gg K$ ). In what follows, we denote the quantized model parameters by  $P_{GB}^{(k)}, P_{BG}^{(k)}, P_E^{(k)}$ ,  $k = 1, \dots, K$ . Finally, we associate each quantization level to a communication quality level. To this end, we compute the average error probability  $\bar{p}_e^{(k)}$  for each quantization level. We have

$$\bar{p}_e^{(k)} = \frac{P_{GB}^{(k)}}{P_{GB}^{(k)} + P_{BG}^{(k)}} P_E^{(k)}, \quad k = 1, \dots, K.$$

The quantization levels are then enumerated such that  $\bar{p}_e^{(1)} < \dots < \bar{p}_e^{(K)}$ . That is, the quantized model parameters  $P_{GB}^{(k)}, P_{BG}^{(k)}, P_E^{(k)}$  correspond to the  $k$ th quality level  $Q_k$ , where  $Q_1$  is the best quality level and  $Q_K$  is the worst quality level. In accordance to [13], we use  $K = 3$  quantization levels in the following. The resulting quantized model parameters and

distance intervals for the three communication quality levels are given in Table I.

TABLE I: Parameters of the simplified range-dependent modified Gilbert model for urban environments.

Quality Parameter	$Q_1$	$Q_2$	$Q_3$
Street layout 1			
Range [m]	0 - 270	271 - 360	361 - 580
$P_{BG}$	0.89	0.40	0.07
$P_{GB}$	0.01	0.23	0.97
$P_E$	0.01	0.88	0.97
Street layout 2			
Range [m]	0 - 330	Impairments only	331 - 580
$P_{BG}$	0.88	0.88	0.16
$P_{GB}$	0.01	0.19	0.47
$P_E$	0.01	0.96	0.80

From Table I, we conclude that the coverage range is essentially equal for the considered urban street layouts. However, for street layout 2, the whole range is represented by the two quality levels  $Q_1$  and  $Q_3$ , while  $Q_2$  is only used to model propagation impairments. This is in contrast to street layout 1, where each of the three quality levels is used in a certain range. Furthermore, we observe that the parameters of  $Q_1$  are almost the same for both street layouts. This is not the case for the quality levels  $Q_2$  and  $Q_3$ , and we show in the following that these differences are important.

### B. Model Validation

The proposed model is clearly most useful if the set of parameters extracted from a particular measurement can be used to describe the link performance for other locations with similar street layouts. To show that this is indeed the case, we first introduce the notion of primary and secondary measurements. The primary measurement is used for the model parameter estimation and the secondary measurement is carried out at a different site. The estimated parameters are then used to model the communication performance at the site of the secondary measurement. By comparing the model output to the secondary measurement we can quantify how well our model describes scenarios with similar street layouts. We note that the type and position of impairments need not be the same in the primary and secondary measurements. Detailed information about the primary and secondary measurements selected for the evaluations in this paper is summarized in Table II.

We next give an example in which the quality levels  $Q_1, Q_2, Q_3$  are used as building blocks to create a site-specific model. Let us assume that we model a type-1 street layout. First, we use the parameters from Table I to get a coarse model for the communication range and quality levels (cf. Fig. 4(a)). In the next step, we refine the model to account for the propagation impairments by adjusting the model parameters where necessary. In our example, there is a 10m wide intersection

TABLE II: Details of primary and secondary measurements.

	RSU position (lat./lon.)	Street name	Driving direction
Street layout 1			
Primary	44.4946 / 11.3143	A. Costa	East
Secondary	44.4947 / 11.3284	A. Costa	East
Street layout 2			
Primary	44.4863 / 11.3397	E. Panzacchi	East
Secondary	44.5012 / 11.3310	S. Porrettana	South-East

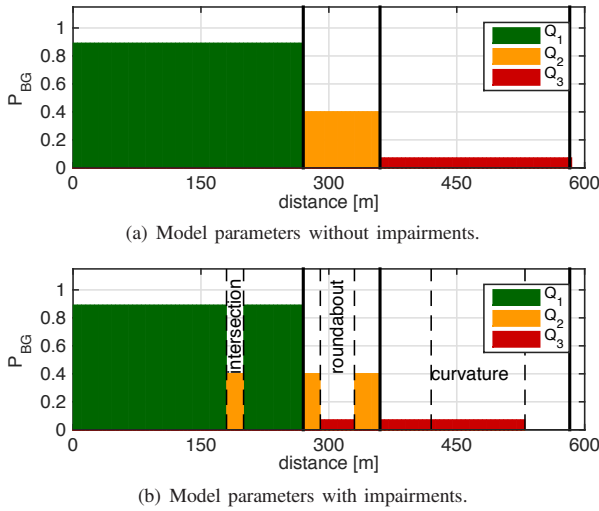


Fig. 4: Example of model parameter adjustment.

starting 185 m away from the transmitter. Since the intersection is located within  $Q_1$ , we change the parameters to  $Q_2$  (cf. Sec. IV-B) in the range 180 – 200 m. Furthermore, a roundabout of 30 m in diameter starts 295 m away from the transmitter. The roundabout is located in the intermediate quality level and we therefore change the parameters from  $Q_2$  to  $Q_3$  in the range 290 – 330 m. Finally, the street along which the vehicle is driving contains a curve at 420 m. Since the street curvature occurs in the unreliable communication range, the parameters remain unchanged. However, the communication range is shortened and thus the coverage ends at 520 m. The resulting model parameters, taking into account the propagation impairments are shown in Fig. 4(b). As soon as the model parameters capturing the street layout and impairments are fixed, packet error traces can be generated using our range-dependent modified Gilbert model.

Packet error traces generated in this manner are shown in Fig. 5 in terms of the packet delivery ratio (PDR) vs. distance. The black solid lines in Fig. 5 show the measured PDR performance and the green dashed lines correspond to the model-generated traces. To emphasize the strong influence of the street layout on the communication quality, and thus on the model parameters, we generated packet error traces using the parameters of the respective other street layout. That is,

TABLE III: Performance indicators.

	Measured reference	Original model	Simplified model	Simplified for other env.
Street layout 1				
KLD	21.9	16.5	18.4	26.9
Erroneous packets [%]	22.6	23.2	22.1	14.4
Street layout 2				
KLD	23.1	17.7	21.8	48.2
Erroneous packets [%]	14.6	15.2	15.5	23.8

for modeling the performance of the secondary measurement site, we used the parameters estimated for the “wrong” street layout. The resulting performance is shown by the dash-dotted red lines in Fig. 5.

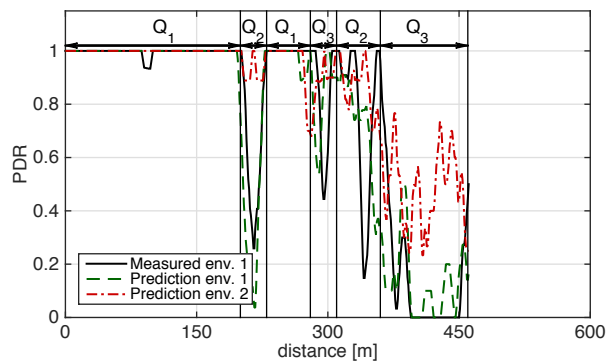
For both street layouts, the green dashed lines are obtained using the proposed modeling approach with parameters of the matching environment. Considering the simplicity of our model, these results describe the measured performance with excellent accuracy. However, this is not the case if the parameters of a different environment are used, i.e., the red dash-dotted lines deviate significantly from the black solid lines.

To quantify this observation, we define two performance indicators: the Kullback-Leibler divergence (KLD) and the percentage of erroneous packets. The KLD can be viewed as a distance measure between two nonnegative vectors. In our setting, the KLD between the PDR vectors  $\text{PDR}_1[n]$  and  $\text{PDR}_2[n]$ ,  $n = 1, \dots, N$ , is given by

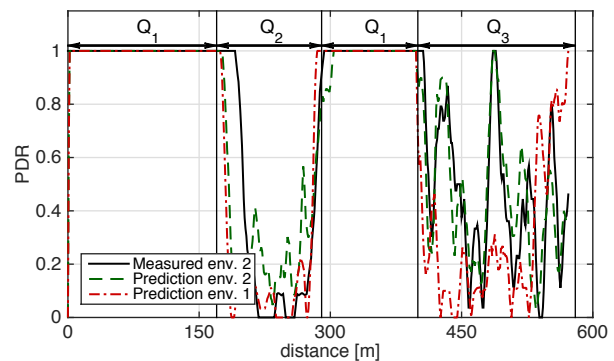
$$\text{KLD}_{1,2} = \sum_{n=1}^N (\text{PDR}_1[n] \log \frac{\text{PDR}_1[n]}{\text{PDR}_2[n]} - \text{PDR}_1[n] + \text{PDR}_2[n]),$$

where  $\text{PDR}_l[n]$  is the PDR of the  $l$ th measurement in the  $n$ th interval. The KLD satisfies  $\text{KLD}_{1,2} \geq 0$  with  $\text{KLD}_{1,2} = 0$  if and only if  $\text{PDR}_1[n] = \text{PDR}_2[n]$  for all  $n$ .

Table III summarizes the performance indicators calculated for both street layouts. In this table “measured reference” for KLD is given by the mean KLD computed between several repetitions of the same measurement. Specifically, we compute all pairwise KLDs between the individual measurement repetitions. That is, for  $M$  measurement repetitions we calculate the arithmetic mean of  $\frac{1}{2} \frac{M!}{(M-2)!}$  KLDs. The percentage of erroneous packets is obtained as the mean percentage of erroneous packets for several repetitions of the same measurement. Clearly, the more accurate our model is, the closer the performance indicators are to the reference values. The performance indicators for “original model” show the performance achievable with the range-dependent modified Gilbert model and unquantized model parameters (cf. [8]). The columns “simplified model” and “simplified for other env.” show the performance indicators for the proposed modeling approach using parameters of the matching (cf. green solid line in Fig. 5) and the “wrong” (cf. red dash-dotted line in Fig. 5) street layout, respectively. All model-generated results are obtained by averaging over 1000 packet error traces.



(a) PDR vs. distance for street layout 1.



(b) PDR vs. distance for street layout 2.

Fig. 5: PDR vs. distance performance. Black solid lines show secondary measurement samples. Green dashed lines represent the model-generated performance with parameters of equivalent environment. Red dash-dotted lines indicate the model-generated performance with parameters of mismatched street layout.

From Table III, we observe that for both street layouts the deviation of the performance indicators from the reference values is small for the original model and also for the proposed simplified model. The KLD shows that the original model resembles the measurements more closely than the simplified model. However, the KLD for the simplified model is still below the average KLD between the individual measurement repetitions. Furthermore, we observe that the simplified model with parameter mismatch differs significantly from the measurements, in particular in terms of the percentage of erroneous packets. These results corroborate the strong influence of the street layout on the communication performance which is captured by the model parameters.

We conclude that the proposed modeling approach is simple and yields accurate results. The proposed model uses only 9 probabilities and enables the generation of realistic packet error traces for the entire coverage range of a certain scenario. Compared to the original range-dependent modified Gilbert model, computational complexity is reduced but some precision is lost due to the vector quantization of the parameters. However, the proposed model is more universal since it allows us to use the quality levels which result from the quantization as building blocks for site-specific models.

## VI. CONCLUSIONS

In this contribution, we proposed a computationally inexpensive modeling approach that allows us to generate realistic packet error patterns for urban environments. The proposed modeling approach suggests to divide the communication range into three quality levels. The packet error performance in each quality level is modeled by a two-state hidden Markov model with three parameters. Four types of street layouts typical for urban environments were identified and the corresponding model parameters for two of them were derived in this contribution. Moreover, the most common types of propagation impairments for urban environments were classified and a way to account for their presence was introduced. A comparison between measured and modeled link

performance shows the accuracy of our model and underpins its ability to account for specific street layouts and propagation impairments.

## ACKNOWLEDGMENTS

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## REFERENCES

- [1] B. Gallagher, H. Akatsuka, and H. Suzuki, "Wireless communications for vehicle safety: Radio link performance and wireless connectivity methods," *IEEE Veh. Technol. Mag.*, vol. 1, no. 4, pp. 4–24, Dec. 2006.
- [2] V. Shivaldova and C. Mecklenbräuer, "Real-world measurements-based evaluation of IEEE 802.11p system performance," in *Proc. IEEE Int. Symp. Wireless Veh. Commun. (WiVeC)*, Jun. 2013.
- [3] V. Shivaldova, A. Paier, D. Smely, and C. Mecklenbräuer, "On roadside unit antenna measurements for vehicle-to-infrastructure communications," in *Proc. IEEE Int. Sym. Personal Indoor and Mobile Radio Commun. (PIMRC)*, Sep. 2012, pp. 1295–1299.
- [4] J. Nuckelt, T. Abbas, F. Tufvesson, C. Mecklenbräuer, L. Bernado, and T. Kurner, "Comparison of ray tracing and channel-sounder measurements for vehicular communications," in *Proc. IEEE Veh. Technol. Conf. (VTC)*, Jun. 2013.
- [5] K. Mahler, P. Paschalidis, A. Kortke, M. Peter, and W. Keusgen, "Realistic IEEE 802.11p transmission simulations based on channel sounder measurement data," in *Proc. IEEE Veh. Technol. Conf. (VTC)*, Sep. 2013.
- [6] M. L. Jakobsen, T. Pedersen, and B. H. Fleury, "Simulation of birth-death dynamics in time-variant stochastic radio channels," in *Proc. Int. Zurich Seminar on Communications*, Feb. 2014, p. 124.
- [7] M. Sepulcre, J. Gozalvez, O. Altintas, and H. Kremo, "Exploiting context information for estimating the performance of vehicular communications," in *Proc. IEEE Veh. Netw. Conf. (VNC)*, Dec. 2013, pp. 39–46.
- [8] V. Shivaldova, A. Winkelbauer, and C. Mecklenbräuer, "Vehicular link performance: From real-world experiments to reliability models and performance analysis," *IEEE Veh. Technol. Mag.*, vol. 8, no. 4, pp. 35–44, Dec. 2013.
- [9] J. Gozalvez, M. Sepulcre, and R. Bauza, "IEEE 802.11p vehicle to infrastructure communications in urban environments," *IEEE Commun. Mag.*, vol. 50, no. 5, pp. 176–183, May 2012.
- [10] FP7 ICT Project iTETRIS, available at: <http://ict-itetris.eu/>.
- [11] E. N. Gilbert, "Capacity of a burst-noise channel," *Bell System Technical Journal*, vol. 39, no. 9, pp. 1253–1265, 1960.
- [12] L. E. Baum, T. Petrie, G. Soules, and N. Weiss, "A maximization technique occurring in the statistical analysis of probabilistic functions of Markov chains," *Ann. Math. Stat.*, vol. 41, pp. 164–171, Feb. 1970.
- [13] V. Shivaldova and C. Mecklenbräuer, "Quantization-based complexity reduction for range-dependent modified Gilbert model," in *Proc. IEEE Sensor Array and Multichannel Signal Process. Workshop (SAM)*, Jun. 2014, pp. 345–348.